

Asymmetry and Long Memory in Volatility Modelling*

Manabu Asai

Faculty of Economics
Soka University, Japan

Michael McAleer

Econometric Institute
Erasmus School of Economics
Erasmus University Rotterdam
and

Tinbergen Institute
The Netherlands

and

Institute of Economic Research
Kyoto University, Japan

and

Department of Quantitative Economics
Complutense University of Madrid, Spain

Marcelo C. Medeiros

Department of Economics
Pontifical Catholic University of Rio de Janeiro, Brazil

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Abstract

A wide variety of conditional and stochastic variance models has been used to estimate latent volatility (or risk). In this paper, we propose a new long memory asymmetric volatility model which captures more flexible asymmetric patterns as compared with several existing models. We extend the new specification to realized volatility by taking account of measurement errors, and use the Efficient Importance Sampling technique to estimate the model. As an empirical example, we apply the new model to the realized volatility of S&P500 to show that the new specification of asymmetry significantly improves the goodness of fit, and that the out-of-sample forecasts and Value-at-Risk (VaR) thresholds are satisfactory. Overall, the results of the out-of-sample forecasts show the adequacy of the new asymmetric and long memory volatility model for the period including the global financial crisis.

Keywords: Asymmetric volatility, long memory, realized volatility, measurement errors, efficient importance sampling.

1 Introduction

The accurate specification and modelling of risk are integral to optimal portfolio selection and risk management using high frequency and ultra high frequency data. In this context, a wide variety of conditional and stochastic variance models has been used to estimate latent volatility (that is, to measure risk) using high frequency data, while the availability of tick data has led to alternative models of realized volatility to estimate integrated volatility in analysing ultra high frequency data (see McAleer (2005) for a comprehensive review of univariate and multivariate, and symmetric and asymmetric, conditional and stochastic volatility models, and Asai, McAleer and Yu (2006) for a detailed review of alternative specifications and estimation algorithms for multivariate stochastic volatility models).

In the framework of diffusion processes, the daily variance of stock return is expressed as an integral of the intraday variance, which is called the integrated variance. If the microstructure noise is ignored, we may estimate the integrated variance by the sum of squared returns of ultra high frequency data. Such an estimator is called the realized variance, which corresponds to an estimate of the integrated variance, namely the true daily variance. In this paper, we refer to the square root of the integrated variance and of the realized variance as the Integrated Volatility (IV) and Realized Volatility (RV), respectively. For a recent extensive review of the RV literature, see McAleer and Medeiros (2008), and Bandi and Renò (2008), Todorov (2009) and Shephard and Sheppard (2010), among others, for more recent developments regarding the modelling and estimation of stochastic volatility using high frequency data.

Recent empirical results from the RV literature show two typical features in volatility, namely the asymmetric effect on volatility caused by previous returns, and the long-range dependence in volatility. The former issue has been investigated by Bollerslev and Zhou (2006), Bollerslev, Litvinova and Tauchen (2006), Bollerslev, Sizova and Tauchen (2011), Chen and Ghysels (2008), Martens, van Dijk and de Pooter (2009), and Patton and Sheppard (2010), among others. With respect to the latter point, the autoregressive fractionally integrated model has been used by Andersen, Bollerslev, Diebold and Labys (2001), Koopman, Jungbacker and Hol (2005) and Pong, Shackelton, Taylor and Xu (2004), among others, while other studies have used the heterogeneous autoregressive model of Corsi (2009) to approximate the hyperbolic decay rates associated with long memory models.

The purpose of the paper is to propose a new specification of the asymmetric and long memory volatility model, which allows flexible patterns in order to capture empirical regularities. Based on the general specification, we examine alternative stochastic volatility models that have recently been developed and estimated. Some of the corresponding SV models are in Harvey and Shephard (1996), Danielsson (1994), and Asai and McAleer (2005, 2011), with similar attempts having been considered by Bollerslev, Sizova and Tauchen (2011), Martens, van Dijk and de Pooter (2009), and Corsi and Renò (2010). Bollerslev, Sizova and Tauchen (2011) develop an equilibrium model with a continuous time long memory process, while our paper takes a discrete-time approach. Compared with Martens, van Dijk and de Pooter (2009) and Corsi and Renò (2010), our model incorporates a more general specification of the asymmetric effect and exact long memory process.

Upon estimating RV by using ultra high frequency data, one of the major problems that has arisen is that of microstructure noise. Several authors have proposed alternative methods for removing the microstructure noise (see, for example, Bandi and Russell (2011), Barndorff-Nielsen, Hansen, Lunde and Shephard (2008), Zhang, Mykland and Aït-Sahalia (2005), and Hansen, Large and Lunde (2008)). Some methods have provided bias-corrected and consistent estimators of the integrated variance, while other methods have not. Recently, Asai, McAleer and Medeiros (2011a) have shown that, even when a bias-corrected and consistent estimator is used, non-negligible measurement errors remain in estimating and forecasting IV.

Barndorff-Nielsen and Shephard (2002) considered the decomposition of RV as the sum of IV and measurement error, which they call the RV error. In other words, RV is considered to be a proxy for IV. With respect to the third of our aims, we propose a new asymmetric model for RV by extending the general asymmetric volatility model, with an additional term to capture RV errors. It should be noted that introducing a correction for measurement error in the RV process renders the true volatility process unobservable. In order to estimate the proposed model, we employ the efficient importance sampling (EIS) ML method proposed by Liesenfeld and Richard (2003, 2006). The EIS evaluates the log-likelihood function of the model, including the latent process, by using simulations, such as the Monte Carlo Likelihood (MCL) technique of Durbin and Koopman (1997). Compared with the MCL method, the EIS method is applicable to various kinds of latent models (see the discussion in Liesenfeld and Richard (2003)).

The remainder of the paper is organized as follows. Section 2 develops a general long-memory asymmetric volatility model, and examines five kinds of asymmetric SV models. By using the structure of asymmetric effects, Section 3 proposes a new model for RV based on correcting for RV errors. Section 4 examines the finite sample properties of the EIS-ML estimator, while Section 5 presents the empirical results for the RV model using S&P500, and evaluates the new specification of asymmetry with respect to goodness of fit, out-of-sample forecasts, and Value-at-Risk (VaR) thresholds. Section 6 gives some concluding remarks.

2 Structure of Asymmetric Volatility Models with Long Memory

In this section, we propose a new asymmetric volatility model, and compare it with stochastic volatility (SV) models that have recently been developed and estimated.

The return process is given by

$$r_t = m_t + V_t z_t, \quad z_t \sim \text{i.i.d.}(0,1),$$

where m_t and V_t are the time-varying mean and volatility processes, respectively, and z_t is the standardized disturbance. We assume that the log-volatility follows an ARFIMA(p, d, q) process,

$$\ln V_{t+1} = \alpha + (1-L)^{-d} \Phi^{-1}(L) \Theta(L) \xi_t, \quad (1)$$

where L is the lag-operator, $\Phi(L)$ and $\Theta(L)$ are the lag polynomials for the AR and MA coefficients, and $(1-L)^d$ is the fractional difference operator. As suggested by Nelson (1990, 1991) for conditional volatility models, the innovation term in the volatility equation plays an important role in considering asymmetry and leverage effects.

We suggest a generalized error, such that

$$\begin{aligned}\xi_t &= \xi_t^* - E(\xi_t^*) + \eta_t, \quad \eta_t \sim N(0, \sigma_\eta^2), \\ \xi_t^* &= \gamma_1 z_t + \gamma_2 |z_t| + \gamma_3 z_t I(0 \leq z_t < \delta) + \gamma_3 \delta I(\delta \leq z_t),\end{aligned}\tag{2}$$

where γ_1 , γ_2 and γ_3 are parameters, and $I(0 \leq z < \delta)$ is the indicator function, which takes the value of one if $0 \leq z < \delta$, and zero otherwise. The first two terms in ξ_t^* play similar roles as in the EGARCH model. As shown in Harvey and Shephard (1996), the negative sign of the coefficient of z_t produces the dynamic relationship between current return and future volatility, which is called the ‘leverage’ effect. Generally, a sufficient condition for univariate SV models to have a leverage effect is that ξ_t is negatively correlated with z_t . For our new model, a negative sign for γ_1 is expected. Hence, $\gamma_1 z_t$ controls the leverage effect in the new model. On the other hand, $\gamma_2 |z_t|$ governs the size effect. When $\gamma_1 = \gamma_3 = 0$, the term makes the log-volatility increase according to the size of the standardized error.

Turning to the last two terms in ξ_t^* , they contribute to capturing asymmetric effects with greater flexibility. Figure 1 shows the relationship between ξ and z , and implies that negative shocks and large positive shocks increase future volatility via ξ , but small positive shocks decrease volatility. Such a phenomenon has recently been observed in Chen and Ghysels (2008) with a semi-parametric method for realized volatility. Recently, Patton and Sheppard (2010) also attempt to explain it by considering the sign of jumps on the realized volatility measure.

We consider five special cases as follows:

Model (i) Equations (1) and (2), with restrictions $\gamma_1 = \gamma_2 = \gamma_3 = 0$.

Model (ii) Equations (1) and (2), with restrictions $\gamma_1 < 0$ and $\gamma_2 = \gamma_3 = 0$.

Model (iii) Equations (1) and (2), with $\gamma_3 = 0$ and z_t replaced by $r_t - m_t$.

Model (iv) Equations (1) and (2), with $\gamma_3 = 0$ and $\gamma_2 |z_t|$ replaced by $\gamma_2 |r_t - m_t|$.

Model (v) Equations (1) and (2), with $\gamma_3 = 0$.

In order to understand these concepts, it is convenient to consider a simple AR(1) model of log-volatility. Setting $d = 0$, $\Phi(L) = 1 - \phi L$, and $\Theta(L) = 1$ in (1), we have

$$\ln V_{t+1} = (1 - \phi)\alpha + \phi \ln V_t + \xi_t. \quad (3)$$

Taking V_t as the latent process for the stochastic volatility, we may find the following correspondence. Model (i) is the basic SV model of Taylor (1982), which is symmetric, as positive and negative shocks to returns have identical effects on future volatility. Model (ii) corresponds to the SV model suggested in Harvey and Shephard (1996), and re-examined by Yu (2005) (see also Asai and McAleer (2009) for a correction of Yu's (2005) news impact function). Model (iii) was proposed in Danielsson (1994), and was estimated in Asai and McAleer (2005). Model (iv) was suggested in Asai and McAleer (2005) to capture both leverage and asymmetric effects. Model (v) adapts the EGARCH model of Nelson (1991) to the SV literature, and was suggested and estimated in Asai and McAleer (2011). In contrast to Model (iii), Model (v) uses the standardized returns in forecasting future volatility, and can capture various types of asymmetric and leverage effects.

As compared with existing models, the new model in (1) and (2) allows log-volatility to

follow the ARFIMA process, and incorporates more flexible asymmetric effects.

3 Model Specification for Realized Volatility

Let $p(t + \tau)$ be the logarithmic price of a given asset at time τ ($0 \leq \tau \leq 1$) on day t ($t = 1, 2, \dots$). We assume that $p(t + \tau)$ follows a continuous time diffusion process,

$$dp(t + \tau) = \mu(t + \tau)d\tau + \sigma(t + \tau)dW(t + \tau), \quad (4)$$

where $\mu(t + \tau)$ is the drift component, $\sigma(t + \tau)$ is the instantaneous volatility (or standard deviation), and $W(t + \tau)$ is a standard Brownian motion. Let r_t be the daily return, defined as $r_t = p(t) - p(t - 1)$. Conditionally on

$$\mathfrak{F}_t \equiv \mathfrak{F}\{\mu(t + \tau - 1), \sigma(t + \tau - 1)\}_{\tau=0}^{\tau=1},$$

which is the σ -algebra (information set) generated by the sample paths of $\mu(t + \tau - 1)$

and $\sigma(t + \tau - 1)$ ($0 \leq \tau \leq 1$), we have

$$r_t \Big| \mathfrak{F}_t \sim N\left(\int_0^1 \mu(t + \tau - 1)d\tau, \int_0^1 \sigma^2(t + \tau - 1)d\tau\right).$$

The term $\int_0^1 \sigma^2(t + \tau - 1)d\tau$ is known as the *integrated variance*, which is a measure of the day- t ex post volatility. The integrated variance is typically the object of interest as a measure of the true daily volatility.

With respect to the model of the instantaneous volatility, there are several specifications, which are called “continuous-time Stochastic Volatility (SV)” models (see Ghysels, Harvey and Renault (1996), for example). Hull and White (1987) allow the squared

volatility to follow a diffusion process:

$$d\sigma^2 = \alpha\sigma^2 d\tau + \omega\sigma^2 dB, \quad (5)$$

where B is a second Brownian motion, and α and ω are parameters. Here, we have omitted $(t + \tau)$ in order to simplify the notation. Hull and White (1987) assume a negative correlation between W and B , thereby incorporating leverage effects. The model in (5) is closely related to the GARCH diffusion, which is derived as the diffusion limit of a sequence of GARCH(1,1) models (see Nelson (1990)).

Wiggins (1987) assumes that the log-volatility follows a Gaussian Ornstein-Uhlenbeck (OU) process:

$$d \log \sigma^2 = \alpha (\mu - \log \sigma^2) \sigma^2 d\tau + \omega dB. \quad (6)$$

In the specification, we may introduce leverage effects by assuming a negative correlation between W and B . The asymmetric SV model of Harvey and Shephard (1996) is considered to be an Euler-Maruyama approximation of the continuous-time model (6), with negative correlation. Three major extensions of such diffusion-based SV models incorporate jumps to volatility process (Eraker, Johannes and Polson (2003)), model volatility as a function of a number of factors (Chernov et al. (2003)), and allow the log-volatility to follow a long memory process (Comte and Renault (1998)).

If the underlying process of the instantaneous volatility is a continuous-time SV model, the resulting integrated variance is still a stochastic process. At this stage, it may be useful to distinguish the differences and similarities among the conditional variance, stochastic variance, and integrated variance. As shown in Nelson (1990), it is possible to consider the diffusion limits of typical conditional variance models, such as the GARCH model and the exponential GARCH model of Nelson (1991). Hence, conditional variance models are considered to be approximations of continuous-time SV models. Alternative approximations are the (discrete-time) SV models of Taylor (1982) and Harvey and Shephard (1996), which are obtained by the Euler-Maruyama discretization of the continuous-time SV models. Compared with the class of GARCH models, discrete-time SV models give better approximations in the sense that the latter can be derived straightforwardly from continuous-time SV models. Therefore, the

conditional and (discrete-time) stochastic variance can be considered as approximations of the integrated variance obtained by continuous-time SV models.

Although the integrated variance is unobservable, it is possible to estimate it using high frequency data. Such estimates are called “Realized Volatility (RV)”. Zhang, Mykland and Aït-Sahalia (2005) and Barndorff-Nielsen, Hansen, Lunde and Shephard (2008) have proposed consistent estimator of the integrated variance, under the existence of microstructure noise (for extensive reviews of the RV literature, see Bandi and Russell (2011) and McAleer and Medeiros (2008)). As observed in Barndorff-Nielsen and Shephard (2002), we can always decompose RV as the sum of IV and a measurement error, which they call the ‘RV error’. According to their analysis, even if we have a consistent estimator of IV, the RV contains a measurement error, which is not negligible.

Now we specify the new asymmetric model for realized volatility (RV), noting the correspondence that $m_t = \int_0^1 \mu(t+\tau-1)d\tau$, $V_t^2 = \int_0^1 \sigma^2(t+\tau-1)d\tau$ and $z_t \sim N(0,1)$.

Assume that the RV is a consistent estimator of integrated volatility (IV). Barndorff-Nielsen and Shephard (2002) refer to the measurement error, defined by the difference between RV and IV, as the RV error. Barndorff-Nielsen and Shephard (2002), Bollerslev and Zhou (2002) and Asai, McAleer and Medeiros (2011a) showed it is useful to employ an ad hoc approach that accommodates an error term with constant variance.

Let y_t be the daily log RV, in which RV is a consistent estimate of IV. The new asymmetric model for RV to be analysed in the paper is given by

$$\begin{aligned}
y_t &= \ln V_t + U_t, \quad E(U_t) = 0, \quad V(U_t) = \sigma_u^2, \\
\ln V_{t+1} &= \alpha + (1-L)^{-d} \Phi^{-1}(L) \Theta(L) \xi_t \\
\xi_t &= \xi_t^* - E(\xi_t^*) + \eta_t, \quad \eta_t \sim N(0, \sigma_\eta^2), \\
\xi_t^* &= \gamma_1 z_t + \gamma_2 |z_t| + \gamma_3 z_t I(0 \leq z_t < \delta) - \gamma_3 \delta I(\delta \leq z_t), \\
z_t &= r_t / V_t,
\end{aligned} \tag{7}$$

where z_t is the standardized return and follows the standard normal distribution. This specification enables U_t to capture the measurement errors in RV. We will refer to this model as the “RV-ARFIMA(p, d, q)-AS($\gamma_1, \gamma_2, \gamma_3$)-noise” model. The model allows various types of symmetric and/or asymmetric effects, long-memory property, and takes account of the realized volatility errors. If the measurement errors are neglected, we will have a special case with $\sigma_u = 0$. It should be noted that we consider the mean subtracted return, r_t , instead of return.

4 EIS-ML Estimation

The likelihood function for the asymmetric model in equation (7) includes high-dimensional integration, which is difficult to calculate numerically. We employ the Efficient Importance Sampling (EIS) method developed by Liesenfeld and Richard (2003, 2006) for evaluating the log-likelihood. A Web-Appendix explains the likelihood evaluation via EIS in detail (see Asai, McAleer and Mediros (2011b)).

The pilot method for the EIS is the Accelerated Gaussian Importance Sampling (AGIS) approach, as developed in Danielsson and Richard (1993). The AGIS approach is designed to estimate dynamic latent variable models, where the latent variable follows a linear Gaussian process. While the AGIS technique has limited applicability, the EIS is applicable to models with more flexible classes of distributions and specifications for the latent variables. As in the case of AGIS, EIS is a Monte Carlo technique for the evaluation of high-dimensional integrals. The EIS relies on a sequence of simple low-dimensional least squares regressions to obtain a very accurate global approximation of the integrand. This approximation leads to a Monte Carlo sampler, which produces highly accurate Monte Carlo estimates of the likelihood.

We present the results of a Monte Carlo study to investigate the small sample performance of the ML estimation procedure via EIS. We generate R simulated time series for RV-ARFIMA($1, d, 0$)-AS($\gamma_1, \gamma_2, \gamma_3$)-noise model in equation (7) and for some given ‘true’ parameter vector θ . Subsequently, we treat θ as unknown and estimate it for each series using the EIS maximum likelihood method. We compute the sample

mean, standard deviation and root mean squared error (RMSE) and compare it with the ‘true’ parameter value.

The ‘true’ parameter values for generating Monte Carlo samples are given in the first column of Table 1, which is obtained by our empirical analysis in Section 5. The results given in Table 1 are for the typical sample size $T = 2500$ with the number of iterations set to $R = 300$. Table 1 shows that most values of the standard deviations are close to those of the RMSE, indicating that the biases in finite samples are negligible.

5 Empirical Results

5.1 Data and Preliminary Results

The empirical analysis focuses on the RV of S&P500. In calculating daily realized volatility, we use the realized kernel estimator with modified generalized Tukey-Hanning weights of order two, according to Barndorff-Nielsen, Hansen, Lunde, and Shephard (2008). We clean the data for outliers, and discard transactions outside trading hours, considering transactions between 9.30am and 4.00pm. Following Barndorff-Nielsen, Hansen, Lunde, and Shephard (2008), we use a 60-second activity fixed tick time sampling scheme so that we obtain the same number of observations each day. The sample period is Jan/3/1996 to Dec/30/2008, giving $T = 3238$ observations of RV. We use the first 2638 observations for estimation and save the last 600 observations for forecasting. The forecasting period includes the global financial crisis.

As a preliminary analysis, we consider the new Fractional Integrated EGARCH- t models given in Section 3, as

$$\begin{aligned}
 r_t &= \sigma_t z_t, \quad z_t \sim St(\nu), \\
 \ln \sigma_{t+1}^2 &= \alpha + (1-L)^{-d} \Phi^{-1}(L) \Theta(L) \xi_t, \\
 \xi_t &= \xi_t^* - E(\xi_t^*), \\
 \xi_t^* &= \gamma_1 z_t + \gamma_2 |z_t| + \gamma_3 z_t I(0 \leq z_t < \delta) - \gamma_3 \delta I(\delta \leq z_t),
 \end{aligned} \tag{8}$$

where $St(\nu)$ denotes the standardized t distribution, with degrees of freedom given by ν . Note that this model implicitly specifies that $\sigma_\eta = 0$, so that σ_t is determined by the past information. We denote this as the FIEGARCH(p, d, q)- t -AS($\gamma_1, \gamma_2, \gamma_3$) model and, for the case $d=0$, as the EGARCH(p, q)- t -AS($\gamma_1, \gamma_2, \gamma_3$) model.

We estimated two kinds of models, namely EGARCH(1,1)- t -AS($\gamma_1, \gamma_2, \gamma_3$) and FIEGARCH(1, d ,1)- t -AS($\gamma_1, \gamma_2, 0$). Table 2 shows the ML estimates of these models, with initial values of 1000. For the former model, all the estimated parameters, except for α and γ_3 , are significant at the five percent level. The estimate of ϕ is close to 0.99, showing high persistence in volatility. The estimate of γ_1 is negative, while that of γ_2 is positive. The estimate of $1/\nu$ is 0.08, indicating that the estimate of ν is close to 13. The results are typical for the EGARCH- t specification. For the long memory model, all the estimated parameters, except for γ_1 and $1/\nu$, are significant. This specification shows the lack of importance of asymmetric effects and heavy-tailed conditional distributions. Both AIC and BIC favour the FIEGARCH(1, d ,1)- t -AS($\gamma_1, \gamma_2, 0$) model. Similar results are also found in the literature for the FIEGARCH- t specification.

5.2 Estimates for RV Models

In the following, we will show that the empirical results for RV models are substantially different from those associated with EGARCH models. It should be noted that it is inadequate to compare the log-likelihood of EGARCH models with that of RV models as the former is based on r_t while the latter is based on RV, namely y_t . Furthermore, the fat tails of the conditional distribution of r_t are irrelevant for the estimation of the RV model.

Table 3 shows the EIS-ML results of the RV-AR(1)-AS($\gamma_1, \gamma_2, \gamma_3$)-noise model. Regarding asymmetry, we consider four specifications, namely AS(0,0,0), AS($\gamma_1, 0, 0$), AS($\gamma_1, \gamma_2, 0$), and AS($\gamma_1, \gamma_2, \gamma_3$). All the estimated parameters are significant at the 5% level. As the AS($\gamma_1, \gamma_2, \gamma_3$) model has the smallest AIC and BIC, we report the empirical results only for this specification.

The estimate of σ_u is close to 0.4, showing that the RV errors are not negligible. The estimate of ϕ is 0.986, while that of σ_η is 0.11, which are typical of SV models. The estimate of γ_1 is negative, while that of γ_2 is positive. Unlike the estimates of the EGARCH model, the estimate of γ_3 is negative and significant. Figure 2 gives the news impact from z_t to $\ln V_{t+1}$, showing that negative shocks and large positive shocks increase future volatility, but small positive shocks decrease volatility.

Table 4 presents the EIS-ML results for the RV-ARFIMA(1, d ,0)-AS($\gamma_1, \gamma_2, \gamma_3$)-noise model. As before, we consider four kinds of asymmetric effects. Both AIC and BIC selected the AS($\gamma_1, \gamma_2, \gamma_3$) model, so we will concentrate the empirical analysis on this model. All the estimated parameters are significant at the five percent level. The estimate of σ_u is close to 0.4, indicating that the RV errors are not negligible. The estimate of d is 0.47, showing that the log-volatility has long memory and is a stationary process. The estimate of ϕ is positive and close to 0.4, which is against the typical value of -0.1 in the RV literature. The difference can be explained by the existence of RV errors, $U_t = y_t - \ln V_t$. As shown in the Monte Carlo experiments of Asai, McAleer and Medeiros (2011a), even minor RV errors can cause bias in the estimates if the RV error is neglected in estimation. The signs of γ_1 , γ_2 and γ_3 are the same as in the case of Table 3. Figure 3 shows the news impact from z_t to $\ln V_{t+1}$.

From Tables 2 and 3, we find that the RV-ARFIMA(1, d ,0)-AS($\gamma_1, \gamma_2, \gamma_3$)-noise model

has the smallest AIC, while BIC chooses the RV-AR(1)-AS($\gamma_1, \gamma_2, \gamma_3$)-noise model.

These tables indicate that having the additional term, γ_3 , significantly improves the goodness of fit of the model.

5.3 Forecasting Analysis

Regarding the RV-ARFIMA(1, d ,0)-AS($\gamma_1, \gamma_2, \gamma_3$)-noise model, we examine the performance of the out-of-sample forecasts using the following four approaches: (i) test for equal forecast accuracy; (ii) test model specification; (iii) test the forecasts of the VaR thresholds; (iv) model selection. The benchmark model is the Leverage Heterogeneous Autoregressive (LHAR) model, as suggested in Corsi and Renò (2010). The LHAR model is based on the Heterogeneous Autoregressive (HAR) model of Corsi (2009), which approximates a long memory process, with an extension regarding the leverage effect. Hence, the LHAR model accommodates both long range dependence and the leverage effect. The LHAR model is given by

$$y_t = \beta_1 + \beta_2 y_{t-1} + \beta_3 (y_t)_{t-5} + \beta_4 (y_t)_{t-20} + \beta_5 r_{t-1} I[r_{t-1} < 0] \\ + \beta_6 (r_t)_{t-5} I[(r_t)_{t-5} < 0] + \beta_7 (r_t)_{t-20} I[(r_t)_{t-20} < 0] + \text{error},$$

where $(y_t)_{t-h}$ denotes the h -horizon normalized realized volatility, defined by

$$(y_t)_{t-h} = \frac{y_{t-1} + y_{t-2} + \dots + y_{t-h}}{h},$$

and $(r_t)_{t-h}$ is defined by the same manner. $I[r < 0]$ is the indicator function which take one if r is negative, and zero otherwise. Unlike Corsi and Renò (2010), we work with the consistent estimator of integrated volatility. A similar model is suggested by Martens, van Dijk and de Pooter (2009). Note that it is possible to include the positive part of heterogeneous returns, but they are usually insignificant.

Fixing the sample size at 2,500, we re-estimated the model and computed one-step-ahead forecasts of log-volatility for the last 600 days.

First, we report the result for the Harvey, Leybourne, and Newbold (1997) modification of the Diebold and Mariano (1995) test of equal predictive accuracy. The new asymmetric and long-memory volatility model is compared against the LHAR model. The test statistic follows the standard normal distribution asymptotically under the null hypothesis of equal accuracy. Table 5 shows the test results, indicating the difference between the two forecasts. The test rejects the null hypothesis at 1% for Volatility and at 5% for Log-volatility, which shows the superiority of the new model.

Second, we test the model specification, based on the Mincer-Zarnowitz regression, namely

$$x_t = a + b\hat{x}_{t|t-1} + e_t, \quad t = 1, 2, \dots, 600$$

where x_t can be the observed RV or log-RV on day t , and $\hat{x}_{t|t-1}$ is the one-step-ahead forecast of x_t on day t . If the model is correctly specified, then $a=0$ and $b=1$. Table 6 shows the estimates of the coefficients and the heteroskedasticity-consistent F test statistics for the joint null hypothesis, regarding the LHAR model and the RV-ARFIMA(1, d ,0)-AS($\gamma_1, \gamma_2, \gamma_3$)-noise model, respectively. With respect to the LHAR model, the F tests in both cases rejected the null hypothesis that the model is correctly specified. However, for the new asymmetric and long-memory model, the F test did not reject the null hypothesis.

Third, we calculated the VaR thresholds, accommodating the filtered historical simulation (FHS) approach, which is an effective method for predicting VaR thresholds (see Kuuster et al. (2006) for some recent studies regarding the FHS approach). In short, the FHS approach estimates the empirical distribution of the standardized returns, then obtains the $100p$ percentiles to compute the $100p$ percent VaR thresholds. In our analysis, each time we estimated the model with 2,500 observations, we computed the $100p$ percentiles of the empirical distribution based on the last 500 observations, discarding the first 2,000 observations. Combined with the one-day-ahead forecasts of log-volatility, we computed the $100p$ percent VaR thresholds.

In order to assess the estimated VaR thresholds, the unconditional coverage and independence tests developed by Christoffersen (1998) are widely used. A drawback of the Christoffersen (1998) test for independence is that it tests against a particular alternative of a first-order dependence. The duration-based approach in Christoffersen and Pelletier (2004) allows for testing against more general forms of dependence but still requires a specific alternative. Recently, Candelon et al. (2011) have developed a more robust procedure which does not need a specific distributional assumption for the durations under the alternative. Consider the “hit sequence” of VaR violations, which takes a value of one if the loss is greater than the VaR threshold, and takes the value zero if the VaR is not violated. If we could predict the VaR violations, then that information may help to construct a better model. Hence, the hit sequence of violations should be unpredictable, and should follow an independent Bernoulli distribution with parameter p , indicating that the duration of the hit sequence should follow a geometric distribution. The GMM duration-based test developed by Candelon et al. (2011) works with the J-statistic based on the moments defined by the orthonormal polynomials associated with the geometric distribution. The conditional coverage test and independence test based on q orthonormal polynomials have asymptotic χ_q^2 and χ_{q-1}^2 distributions under their respective null distributions. The unconditional coverage test is given as a special case of the conditional coverage test with $q = 1$.

Table 7 shows the percentage of VaR violations and test results for the LHAR model and new asymmetric and long-memory volatility model, respectively. For both the LHAR model and RV-ARFIMA(1, d ,0)-AS($\gamma_1, \gamma_2, \gamma_3$)-noise model, the tests did not reject the null hypothesis for the 5% and 1% VaR thresholds, indicating that the estimated VaR thresholds are satisfactory. We also conducted the unconditional coverage, independence and conditional coverage tests developed in Christoffersen (1998), and the results are unchanged.

Finally, we select the forecasts using the following MZ equation:

$$x_t = a + b_1 \hat{x}_{t|t-1}^{AS} + b_2 \hat{x}_{t|t-1}^{LHAR} + e_t, \quad t = 1, 2, \dots, 600$$

where $\hat{x}_{t|t-1}^i$ ($i = AS, LHAR$) is the one-step-ahead forecast of x_t on day t , based on the RV-ARFIMA(1, d ,0)-AS($\gamma_1, \gamma_2, \gamma_3$)-noise model (AS) and the LHAR model. We select the forecasts by the conventional t test. As before, we consider two dependent variables, namely volatility and log-volatility. Table 8 gives the results. In both cases, the coefficients of $\hat{x}_{t|t-1}^{LHAR}$ are insignificant, indicating that the data prefer the forecasts of the RV-ARFIMA(1, d ,0)-AS($\gamma_1, \gamma_2, \gamma_3$)-noise model. Overall, the results of the out-of-sample forecasts favour our new asymmetric and long memory volatility model.

6 Concluding Remarks

We proposed a new asymmetric and long-memory volatility model. Regarding the leverage effect, the new model sensitively captures the effects of both large and small, and positive and negative, shocks. Based on the new specification, this paper examined alternative univariate volatility models that have recently been developed and estimated.

We extended the specification of asymmetric and long memory volatility in order to model RV by taking account of the RV errors. This is a general model which includes not only various kinds of asymmetric effects, but also short and long memory specifications. We applied the EIS-ML method to estimate the model of RV, and reported the results for a Monte Carlo experiment.

The empirical results for the RV of S&P500 showed the existence of RV errors. The estimates of the short and long memory models supported the new specification of asymmetric effect, which satisfies the following three conditions: (i) negative shocks to returns increase future volatility; (ii) large positive shocks to returns increase future volatility, but a negative shock has a larger effect on volatility than does a positive shock of equal magnitude; and (iii) small positive shocks to returns decrease future volatility. Overall, the new specification of asymmetry significantly improved the goodness of fit, and the out-of-sample forecasts and VaR thresholds were satisfactory.

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**Table 1: Monte Carlo Results for the EIS-ML Estimator for
RV-AR(1)-AS($\gamma_1, \gamma_2, \gamma_3$)-noise Model**

Parameters	True	Mean	Standard deviation	RMSE
d	0.4727	0.4588	(0.0366)	[0.0392]
ϕ	0.4373	0.4453	(0.0655)	[0.0660]
σ_η	0.1739	0.1792	(0.0156)	[0.0165]
α	0.0046	-0.0018	(0.0019)	[0.0067]
γ_1	-0.0274	-0.0160	(0.0156)	[0.0193]
γ_2	0.0511	0.1179	(0.0155)	[0.0686]
γ_3	-0.2428	-0.2954	(0.0541)	[0.0754]
δ	0.8841	0.8413	(0.1262)	[0.1332]
σ_u	0.3858	0.3845	(0.0089)	[0.0090]

Table 2: ML Estimates of the New EGARCH Class

Parameters	New EGARCH- t		FIEGARCH(1, d ,0)- t	
d			0.4067	(0.0256)
ϕ	0.9877	(0.0038)	-0.2651	(0.0450)
α	0.1397	(0.4560)	0.3101	(0.0777)
γ_1	-0.0936	(0.0138)	0.0115	(0.0441)
γ_2	0.0670	(0.0204)	2.4407	(0.0776)
γ_3	-0.0783	(0.0967)		
δ	0.7259	(0.4408)		
$1/\nu$	0.0784	(0.0191)	0.0004	(0.0410)
Log-Like	-2419.93		-1721.05	
AIC	4853.86		3454.09	
BIC	4892.31		3489.71	

Note: Standard errors are in parentheses. The first 1,000 observations are used for the initial values for the FIEGARCH- t model.

Table 3: EIS Estimates of RV-AR(1)-AS($\gamma_1, \gamma_2, \gamma_3$)-noise Model

Parameters	AS(0,0,0)	AS($\gamma_1, 0, 0$)	AS($\gamma_1, \gamma_2, 0$)	AS($\gamma_1, \gamma_2, \gamma_3$)
ϕ	0.9747 (0.0051)	0.9728 (0.0040)	0.9870 (0.0044)	0.9856 (0.0044)
σ_η	0.1478 (0.0091)	0.1111 (0.0075)	0.1110 (0.0074)	0.1103 (0.00720)
α	-0.3148 (0.1091)	-0.1795 (0.0788)	-0.8439 (0.3227)	1.2246 (0.5127)
γ_1		-0.0681 (0.0046)	-0.0649 (0.0043)	-0.0418 (0.0062)
γ_2			0.0424 (0.0074)	0.0561 (0.0079)
γ_3				-0.1934 (0.0471)
δ				0.4902 (0.0605)
σ_u	0.4054 (0.0073)	0.4092 (0.0067)	0.4125 (0.0067)	0.4116 (0.0067)
Log-Like	-1921.94	-1821.52	-1806.24	-1793.51
AIC	3851.88	3653.04	3624.48	3603.03
BIC	3875.63	3682.72	3660.10	3650.52

Note: Standard errors are in parentheses.

Table 4: EIS Estimates of RV-ARFIMA(1, d ,0)-AS($\gamma_1, \gamma_2, \gamma_3$)-noise Model

Parameters	AS(0,0,0)	AS($\gamma_1, 0, 0$)	AS($\gamma_1, \gamma_2, 0$)	AS($\gamma_1, \gamma_2, \gamma_3$)
d	0.4955 (0.0039)	0.4987 (0.00089)	0.4748 (0.0090)	0.4727 (0.0076)
ϕ	0.3261 (0.0603)	0.3676 (0.0438)	0.4166 (0.0538)	0.4373 (0.0291)
σ_η	0.2416 (0.0225)	0.1750 (0.0147)	0.1852 (0.0157)	0.1739 (0.0080)
α	-0.5832 (0.2394)	0.0021 (0.0026)	0.0051 (0.0027)	0.0046 (0.0020)
γ_1		-0.0865 (0.0061)	-0.0827 (0.0063)	-0.0275 (0.0075)
γ_2			0.0226 (0.0077)	0.0511 (0.0076)
γ_3				-0.2428 (0.0287)
δ				0.8841 (0.0196)
σ_u	0.3648 (0.01208)	0.3844 (0.0081)	0.3827 (0.0085)	0.3858 (0.0067)
Log-Like	-1908.27	-1819.30	-1811.37	-1792.16
AIC	3826.54	3650.60	3636.75	3602.31
BIC	3856.22	3686.22	3678.30	3655.74

Note: Standard errors are in parentheses.

Table 5: HLN Tests for Equal Forecast Accuracy

HLN	Test Stat.	P-value
Volatility	2.7068	0.0067
Log-Volatility	1.9920	0.0464

Note: HLN is the test for equal forecast accuracy of Harvey, Leybourne, and Newbold (1997), where the new asymmetric volatility model is compared with LHAR. The test statistic follows the standard normal distribution asymptotically under the null hypothesis of equal accuracy.

Table 6: Tests for Model Specification by the MZ Equation

$$x_t = a + b\hat{x}_{t|t-1} + e_t$$

Model	LHAR		RV-ARFIMA(1,d,0)-AS($\gamma_1, \gamma_2, \gamma_3$)-noise	
Dependent variable	Volatility	Log-Volatility	Volatility	Log-Volatility
Constant	0.3128 (0.0378)	-0.0887 (0.0181)	0.0624 (0.0406)	0.0089 (0.0139)
Forecast	0.6198 (0.0426)	0.7088 (0.0197)	0.9891 (0.0531)	0.9547 (0.0280)
<i>F</i> test	80.161 [0.0000]	218.16 [0.0000]	4.8618 [0.0879]	5.3616 [0.0685]

Note: Heteroskedasticity-consistent standard errors are in parentheses, and *p*-values are in brackets. ‘*F* test’ denotes the value of the heteroskedasticity-robust *F* test for the null hypothesis, $H_0 : a = 0, b = 1$.

Table 7: Backtesting VaR Thresholds

Model	LHAR				RV-ARFIMA(1,d,0)-AS($\gamma_1, \gamma_2, \gamma_3$)-noise			
VaR	% Violation	UC	IND	CC	% Violation	UC	IND	CC
5%	0.0583	1.4112 [0.2349]	2.4731 [0.5201]	3.2298 [0.6646]	0.0533	0.2758 [0.5995]	6.5637 [0.1662]	6.4772 [0.2625]
1%	0.0150	0.7092 [0.3997]	10.692 [0.3710]	4.2677 [0.5116]	0.0117	0.0073 [0.9319]	1.9170 [0.7601]	1.8677 [0.8671]

Note: ‘% Violation’ denotes the percentage of days when returns are less than the VaR threshold. UC, IND and CC are the GMM duration-based tests for unconditional coverage, independence and conditional coverage, developed by Candelon et al. (2011). The number of orthonormal polynomials is set to 5. *P*-values are in brackets.

Table 8: Model Selection by MZ Equation

$$x_t = a + b_1 \hat{x}_{t|t-1}^{AS} + b_2 \hat{x}_{t|t-1}^{LHAR} + e_t, \quad t = 1, 2, \dots, 600$$

Dependent Variable	Const	$\hat{x}_{t t-1}^{AS}$	$\hat{x}_{t t-1}^{LHAR}$
Volatility	0.0701 (0.0405)	0.8692 (0.0995)	0.0940 (0.0700)
Log-Volatility	0.0056 (0.0138)	0.9074 (0.0468)	0.0423 (0.0390)

Note: Heteroskedasticity-consistent standard errors are in parentheses.

Figure 1: News Impact from Z_t to ξ_t

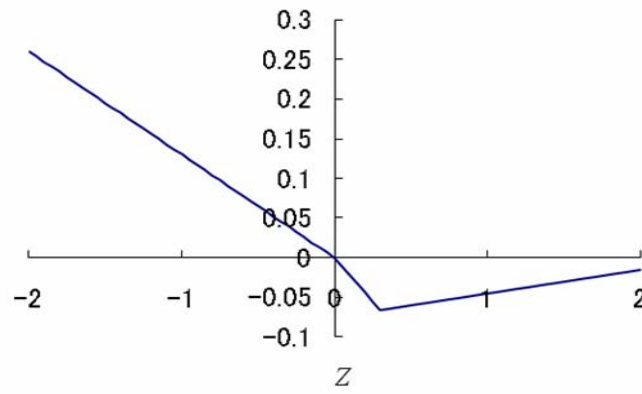


Figure 2: News Impact from Z_t to $\ln I_{t+1}$;
RV-AR(1)-AS($\gamma_1, \gamma_2, \gamma_3$)-Noise Model

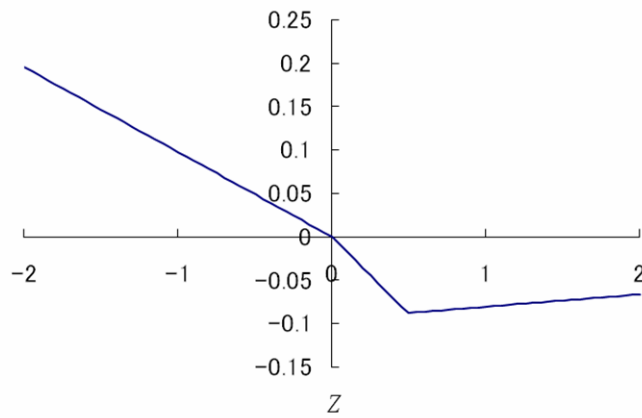


Figure 3: News Impact from Z_t to $\ln I_{t+1}$;
RV-ARFIMA(1, $d,0$)-AS($\gamma_1, \gamma_2, \gamma_3$)-Noise Model

